

Find the smooth minimizer $y(x)$ of the functionals:

1. $J[y] = \int_0^2 (y'(x))^3 y(x) \, dx$, with $y(x)$ satisfying $y(0) = 1, y(2) = 8$.
2. $J[y] = \int_1^2 y'(x)(1 + x^2 y'(x)) \, dx$, $y(1) = 3, y(2) = 5$.
3. $J[y] = \int_0^1 \{(y')^2 + y^2 - 2e^x y\} \, dx$, $y(0) = 0; y(1) = 0$.